

## 1.1 Trading Places

### 1.1.1 Investigate: Jumping Frogs

**Focus questions:** What is the minimum number of moves? What is it about the structure that makes it true?

#### Suggestions

- When you work in groups or as a class, you may find that others approached the problem differently. When this happens, try to find how your approach relates to theirs.
- Record your work including
  - any specific cases you tried,
  - any patterns, procedures, or formulas that you uncovered,
  - any conjectures you made and how you justified them,
  - any questions you still have about your investigation.

Try to solve this fun puzzle.



Six frogs are sitting on lily pads. Three frogs face toward the right, three to the left with a single empty lily pad separating the two groups. They want to trade places so that the group on the left ends up on the right and vice-versa. Only two types of moves are allowed. Each frog can either

- slide from one lily pad (square) to an empty lily pad next to it, or
- jump over a frog of the other group if there is an empty lily pad behind it.

How many moves would it take? What if more frogs come along?

1.1.2 Generalize: Frogs Rule!

**Focus questions:** What is the minimum number of moves for any number of frogs on each side? What is it about the structure that makes it true?

Create a rule that can be used to determine the minimum number of moves if you know the number of frogs on each side. Explain why your rule works.

1.1.3 Extend: Reasoning Algebraically

1. Ben patiently solved the puzzle with the same number of frogs on each side. It took him 120 moves. How many frogs were on each side in his puzzle? Assume he did not make any mistakes.
2. Jonathan claimed that he solved the puzzle in only 31 moves using half of the total number of frogs as Ben did. How is this possible?

1.1.4 Reflect

1. List and describe the patterns you and your classmates observed while solving the puzzle. Explain why these patterns exist.
2. What was the least number of moves you needed to solve the original puzzle (with 3 frogs on each side)? What is it about the structure of the puzzle that makes it true?
3. For any number of frogs on each side, what is the least number of moves needed to solve the puzzle? What is it about the structure of the puzzle that makes it true? What if you have a different number of frogs on each side?
4. What made the puzzle difficult to solve? How did you overcome this difficulty?
5. If you are allowed to make a change in the puzzle, what would you change? How does that change affect the solution?